Educational trends in robotics and control

Integration of math, engineering and emerging AI topics

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Outline

1. Why one needs to review programs in robotics and control now
   - General expectations from EDU programs have changed
   - Economic factors are affecting EDU programs
2. Materials used for analysis
   - Private communication with colleagues-in-charge
   - Web resources
3. MSc Robotics Program Sketch
4. Discovering Robot Abilities
   - Ability Representations and Classes of Motions
   - Example: Natural Sit-Down/Chair-Rise Motions for Humanoid Robot

Why one needs to review programs in robotics and control now

General points

Over the past 15 years, research/technological agenda and best practices have slowly drifted to and explored

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- New problems in robotics that seem to be solvable.
- These problems are important in applications.
- These problems are indeed new and require new scalable arguments
- These problems become motivating for students to enter the topic

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Each of components is constantly developing and evolving at different pace!

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- Often grad students are supported and do not pay fees
- PhD students are supported and do not pay fees
- Grants dictate the shape of research groups: Prof, Post-Doc, few PhD-students, 4-8+ grad and undergrad students

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Main reasons:

- Ambitious students are interested in academic career!

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- Students are looking for a **creative academic environment** and ready to travel!

Materials used for analysis

Universities and Professors

- Northwestern University, ME: Dr. **Kevin Lynch**, Head of ME, Editor-in-Chief IEEE Transactions on Robotics
- University of Michigan, Robotics Institute: Dr. **Jessy Grizzle**, Director
- Michigan State University, EE: Dr. **Hassan Khalil**, Distinguished Professor
- University of Waterloo, CS (CA): Dr. **Chris Nielsen**, Director of graduate studies
- Penn State University: Dr. **Mark Latash**, Distinguished Professor
- MIT, ME: Dr. **Neville Hogan**, Distinguished Professor
- ...

Web resources

- **ABET** (`www.abet.org`): *At ABET, our purpose is to assure confidence in university programs in STEM (science, technology, engineering and mathematics) disciplines...* **We accredit college and university programs** *in the disciplines of applied and natural science, computing, engineering and engineering technology at the associate, bachelors and masters degree levels ... ABET is a nonprofit, non-governmental organization ...*
- **Univ. of Michigan, Robotics Institute** (`robotics.umich.edu`): *Michigan is producing tomorrows robotics leaders.* **Our program is already No. 2 in the nation** *... Students design, create, analyze, and use embodied computational systems that interact with the physical and human environment. They study robotics and its place in the world, drawing on many fields of engineering, including computer science, mechanical engineering, artificial intelligence, computer vision, electrical engineering, control systems...*

Personal experience in lecturing

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- 2018: Royal Institute of Technology (KTH), Sweden
- 2018: Dutch Institute of Systems and Control (DISC)
- 2018-2019: Laboratoire dInformatique, de Robotique et de Microelectronique de Montpellier , Montpellier University and CNRS
- 2018: Doctoral school in Information Technology and Electrical Engineering, Universita degli Studi di Napoli Federico II, Italy

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- 2018-2019: Innopolis University, Russia
- 2019: Lund University, Sweden (tentatively agreed)
- 2019: Moscow State University, Russia (tentatively agreed)

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    1. Recognize an object location by a camera and grasp it
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- **Semester 4**
  - MSc thesis work

MSc Robotics Program Principles

- Each faculty member runs one (or two) elective classes
  1. Robot design, modeling and calibration
  2. Linear control techniques for robotics
  3. Computer vision
  4. Nonlinear control techniques for robotics
  5. Principles of biologically inspired robotics
  6. Robotic tools for grasping and manipulation
  7. Computational tools for robotics
  8. Statistical tools, probabilistic robotics and sensor fusion
  9. AI-tools...

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- Crash course (Semester 1) is compulsory, run by 2-4 faculties and
  1. Each team (3-4 students) generates reports for evaluation
  2. For each of three tasks the student team members are new
  3. The evaluation includes physical experiments and competition between teams

Discovering Robot Abilities

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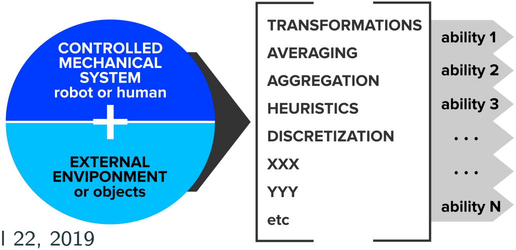

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- **How** can the abstracted action **plan be mapped** to a sequence of corresponding **movement** primitives of a given robotic system?

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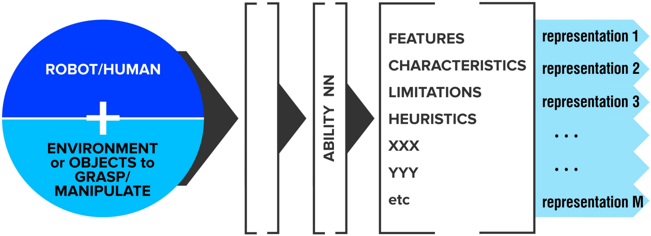
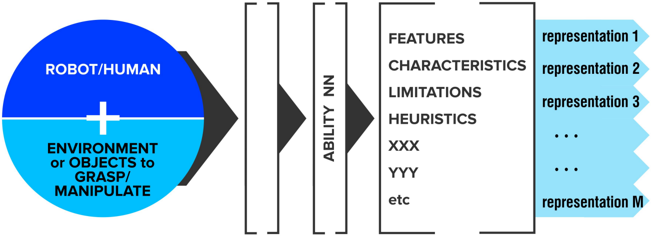
- **How** can the abstracted action **plan be mapped** to a sequence of corresponding **movement** primitives of a given robotic system?
- Learning processes for humans are embodied, **what do AI tools suggest for a given robot?**

AI tools: representing a robot Ability



AI tools: representing a robot Ability

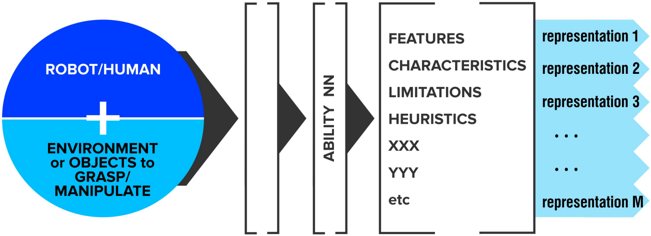


Aggregation, merging . . . techniques in developing AI-models rely on

representations of robot actions or robot abilities . . .

negating time and converting it into order/events/state triggers

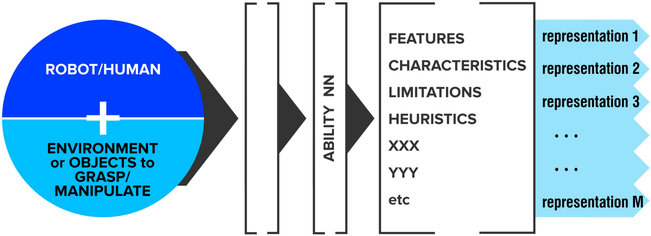
AI tools: representing a robot Ability



Representations (or features) of a given robot ability can be

- **Parametric**: what are a mass/inertia of object, conditions of contact, etc.?

AI tools: representing a robot Ability



Representations (or features) of a given robot ability can be

- **Coordinates dependent**: how many of variables are important and involved in its description?

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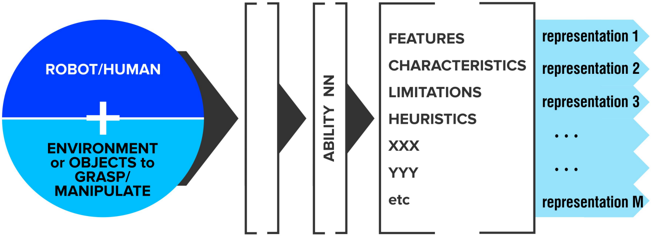
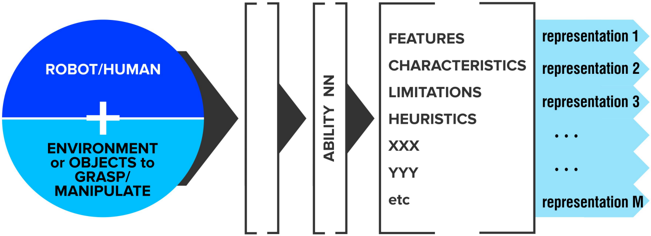


Representations (or features) of a given robot ability can be

- **Constraint dependent**: which physics driven constraints (holonomic, non-holonomic, unilateral) are active for a behavior?

AI tools: representing a robot Ability



Representations (or features) of a given robot ability can be

- **Motion dependent**: ranges, monotonicity, max values for coordinates, velocities, accelerations, control torques for a feasible behavior?

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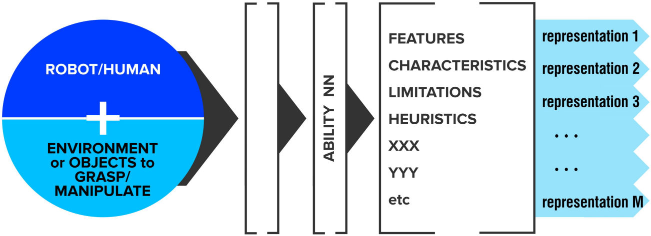
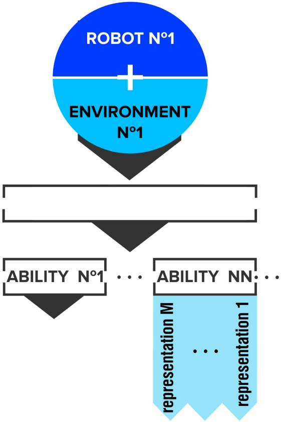
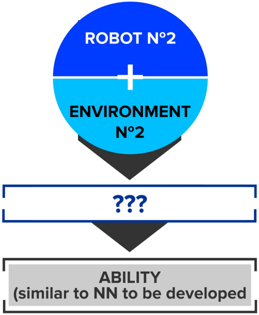


Representations (or features) of a given robot ability can be

- **Feedback dependent**:
  - what quantities are measured?
  - what variables are actuated?
  - what are **invariants induced by feedback**? ...

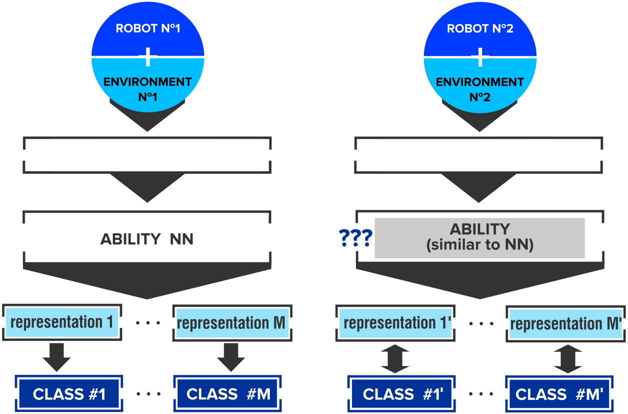
AI tools: discovery of new Ability





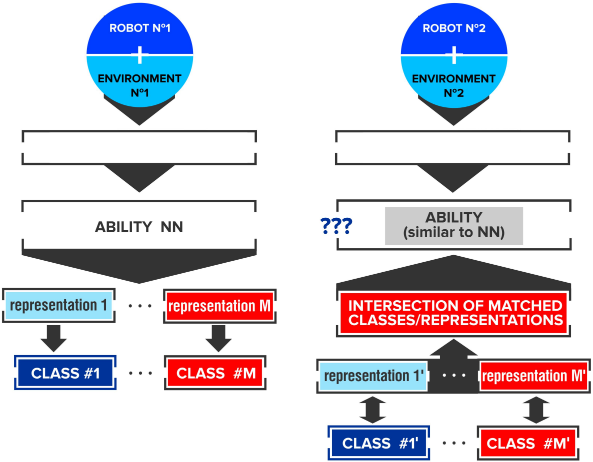
How to find a similar Ability for new robot or new settings?

AI tools: discovering new Abilities



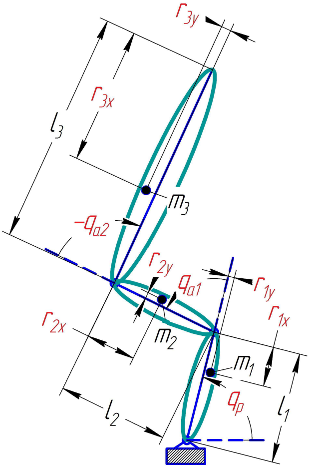
Representation No. $m \Rightarrow$ **Class of behaviors**

AI tools: discovering new Abilities



$$\bigcap_{m\in\Omega}\left\{\text{Class of behaviors of } \textbf{Repres. No. } m\right\} \supset \textbf{New Ability}$$

Representations of Recorded Motion of a Human



- Ankle joint is passive
- Knee/hip joints are actuated
- Hands/fingers/head/back flexibility dynamics are all neglected
- For 3 dof $q = [q_p;\, q_{a1};\, q_{a2}]$ the dynamics are

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \begin{bmatrix} 0 \\ \tau_1 \\ \tau_2 \end{bmatrix}$$

- For recorded motions we searched for the feedback induced invariants

$$q_{a1}(t) = \phi_1(q_p(t)), \quad q_{a2}(t) = \phi_2(q_p(t))$$

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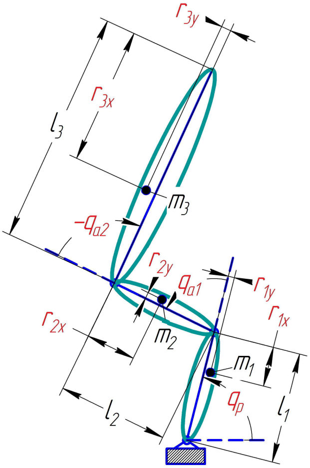

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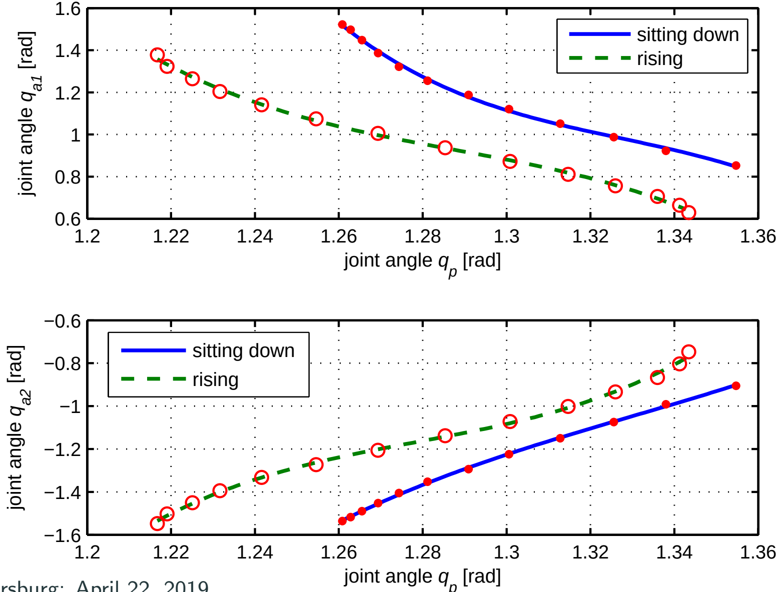
Feedback induced invariants for recorded motions



Feedback induced invariants for recorded motions

The following relations approximate well the sit-down motion

$$q_{knee} = \phi_1(q_p) \approx -868.40 q_p^3 + 3457.5 q_p^2 - 4593 q_p + 2036.7$$

$$q_{torso} = \phi_2(q_p) \approx 269.9 q_p^3 - 1077.1 q_p^2 + 1438.7 q_p - 644.2$$

Here q_p is the ankle angle.

We can differentiate the relations

$$\dot{q}_{knee} = \phi_1'(q_p)\dot{q}_p, \quad \ddot{q}_{knee} = \phi_1''(q_p)\dot{q}_p^2 + \phi_1'(q_p)\ddot{q}_p,$$

The (passive) dynamics will define the decoupled diff. equation for the variable $q_p(t)$ as

$$\alpha(q_p)\ddot{q}_p + \beta(q_p)\dot{q}_p^2 + \gamma(q_p) = 0$$

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We can substitute into the first (passive) equation of dynamics

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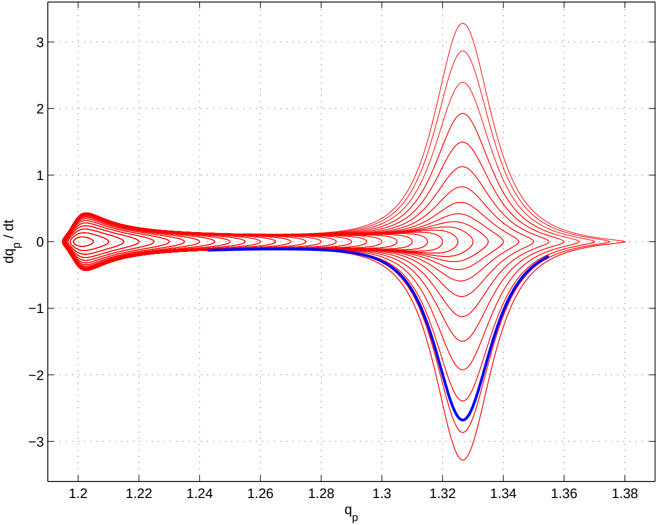
The phase portrait of motion generator dynamics



The blue curve corresponds to the recorded human behavior

Human-Like Features for the Sit-Down Motion

- The reduced system has no equilibrium on the interval $[\min q_p, \max q_p]$ — the range of q_p along the motion;

- For human sit-downs q_p is a piece of periodic trajectory of the reduced system;

- For the human motion q_p has several distinctive sub-regions in a sequel:
  - first, q_p accelerates reaching in short time the minimum value of angular velocity,
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Solutions of the reduced dynamics with small offset exhibit similar behaviors for approximately the same time interval

Developing a Behavior for Humanoid Robot

- Ankle joint is passive
- All other joints are actuated
- For N-degrees of freedom

$$q = [q_p, q_{a_1}, \cdots, q_{a_{N-1}}]$$

the robot dynamics are

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \begin{bmatrix} 0 \\ \tau_1 \\ \vdots \\ \tau_{N-1} \end{bmatrix}$$

- We search for $(N-1)$-feedback invariants

$$q_{a_1} = \phi_1(q_p), \ \ldots, \ q_{a_{N-1}} = \phi_{N-1}(q_p)$$

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Features of a searched motion of a robot

- the ankle angle decreases monotonically from 1.5 to 1 rad;
- the knee angle grows monotonically from 0.4 to 1.6 rad;
- the hip angle decreases monotonically from -0.4 to -1.6 rad;
- control torque for the ankle joint along a motion is zero
- control torque for the knee joint is limited to ± 75 (Nm);
- control torque for the hip joint is limited to ± 80 (Nm);
- time to complete the motion is ≈ 0.5 sec

Features of a searched motion of a robot:

Introduce a family of relations

$$q_{ai} = k_{0i} + k_{1i} \cdot q_p + k_{2i} \cdot q_p^2 + k_{3i} \cdot q_p^3, \qquad i = 1, \ldots, N-1$$

and identify coefficients k_{ij} to meet the specifications

Lemma: The absence of equilibria of

$$\alpha(q_p, k_{ij})\ddot{q}_p + \beta(q_p, k_{ij})\dot{q}_p^2 + \gamma(q_p, k_{ij}) = 0$$

for $q_p \in [1, 1.5]$ [rad] is equivalent to two conditions:

$$\gamma(q_p, k_{ij}) \neq 0 \quad \text{and} \quad \alpha(q_p, k_{ij}) \neq \infty$$

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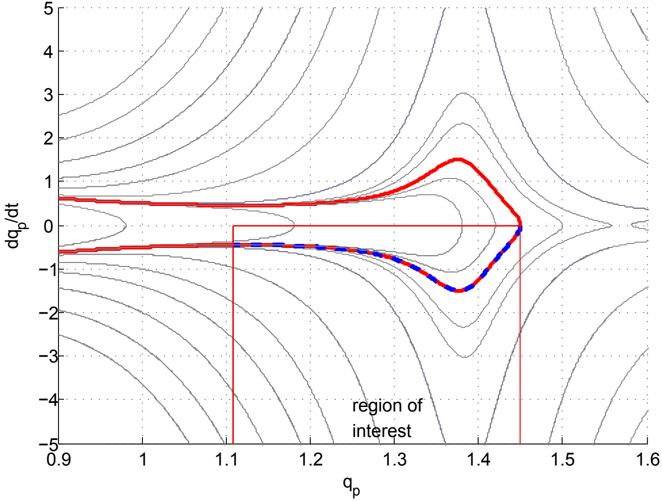
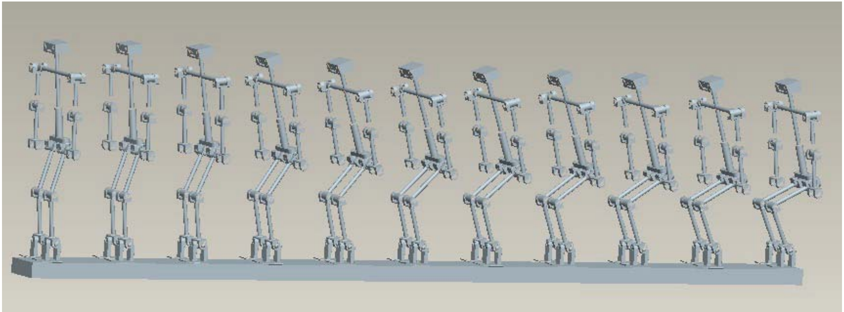


The phase portrait of the reduced system with one of choices of k_{ij}.

A robot sit-down motion



Snap-shots of one of reconstructed motions performed in 0.55 sec

Concluding remarks

- Learning/performing interaction abilities of robots is challenging
- Learning under constraints (underactuation) requires preliminary steps in
  - representation of behaviors
  - alternative choices (movement dependent) of generalized or excessive coordinates
  - searching convenient sets of transverse coordinates (movement dependent)
- Most of arguments are scalable:
  - Cascaded representation for motion planning/control

$$\theta(t) \;\rightarrow\; [q_1(t), \ldots, q_n(t)] = [\Phi_1(\theta(t)), \ldots, \Phi_n(\theta(t))]$$

  - Model based representations for analysis and control:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = B(q)u + \sum_i F_i \;\rightarrow\; q(t)$$

Educational perspectives:

New emerging topics in

- **Modeling** of dynamics of (underactuated) robots

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- **Modeling** of dynamics of (underactuated) robots
- Model based **motion representation** and AI tools
- Model based **motion planning** for (underactuated) robots
- Model based **motion control** for (underactuated) robots
- Developing computational benchmarks, algorithms and labs